

THERMODYNAMIC ANALYSIS OF THE REASONS FOR THE DIFFERENCE  
 BETWEEN THE VALUES OF THE THERMAL CONDUCTIVITIES OF GASES  
 AS MEASURED BY STEADY-STATE AND TRANSIENT METHODS

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It is shown that the effective thermal conductivities of a gas measured by steady-state and transient methods are not equal.

The thermal conductivities  $\lambda$  of gases and gas mixtures in the 90-6000°K range are measured both by steady-state and by transient methods. At temperatures of 90-1500°K, as a rule  $\lambda$  is measured by steady-state methods; however, recently there have appeared experimental data on the thermal conductivities of a number of gases measured by a transient hot-wire method [1-3]. In the 1500-6000°K range  $\lambda$  is measured by a transient shock-tube method [4, 5].

The transient process of heat transfer in a single-component gas is characterized by the presence of a pressure gradient (density gradient). The presence of the latter in the measuring system results in an additional contribution to the conductivity value. Therefore in principle, data obtained by steady-state transient methods must differ in value, and this is confirmed by a comparison of the experimental results [6-8]. As can be seen from Figs. 1 and 2, the data on the thermal conductivities of gases determined by the transient hot-wire method (Fig. 1) lie above the data obtained by steady-state methods.

Let us analyze the reasons for the difference. We write an expression for the heat flux in a single-component gas when there is a pressure gradient in the system (which corresponds to the case considered in [9]):

$$\vec{I}_q = L_{21}\vec{X}_M + L_{22}\vec{X}_q = -\lambda^H \nabla T, \quad (1)$$

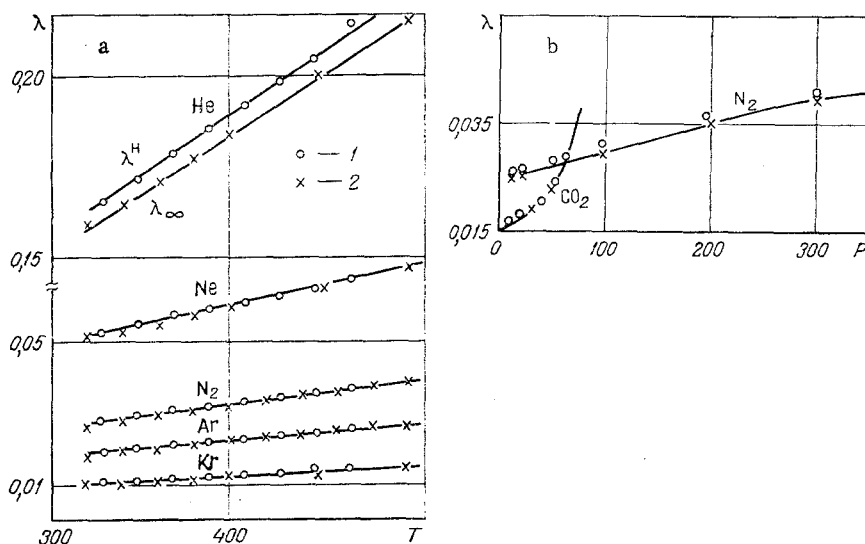


Fig. 1. Thermal conductivities of gases as functions of temperature (a) and pressure (b): 1) transient hot-wire method; 2) steady-state method; the points represent the data of [2, 3, 15].  $\lambda$  in W/m·°K.

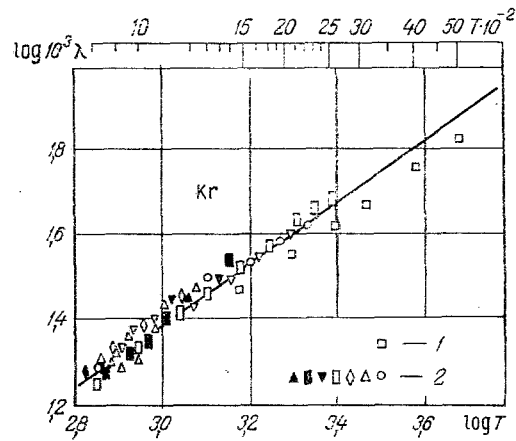


Fig. 2. Thermal conductivity of krypton as a function of temperature [6]: 1) thermal conductivity measured by the shock-tube method; 2) measured by stationary methods.

since the volume of the system is constant:

$$\vec{X}_M = -\frac{k}{M} \left( \frac{\nabla \ln p}{\nabla \ln T} - 2.5 \right) \frac{\nabla T}{T}, \quad \vec{X}_q = -\frac{\nabla T}{T^2}.$$

It can be shown that the relation for the effective thermal conductivity  $\lambda^n$  has the form

$$\lambda^n = \lambda_\infty \left( 1 + \frac{1}{\tilde{Q}^{*-1} - 2.5} \frac{\nabla \ln p}{\nabla \ln T} \right), \quad (2)$$

where

$$\lambda_\infty = \frac{L_{22}}{T^2} (1 - 2.5\tilde{Q}^*); \quad \tilde{Q}^* = \frac{L_{21}}{L_{22}} \frac{k}{M} T.$$

According to (2),  $\lambda^n$  differs from  $\lambda_\infty$  by an amount proportional to  $\nabla \ln p / \nabla \ln T$ , i.e.,  $\lambda^n > \lambda_\infty$ , since  $\nabla p$  and  $\nabla T$  are taken in the same direction (nonstationary hot-wire method).

We write an expression for the production of entropy in a nonstationary heat-transfer process in a single-component gas. By the second law of thermodynamics,  $\sigma_s \geq 0$ , i.e.,

$$\sigma_s = -\vec{I}_q \cdot \frac{\nabla T}{T^2} \geq 0. \quad (3)$$

If the volume of the system is constant, the flux-density vector in the nonstationary state is determined by the relation  $\vec{I}_q = -\lambda_\infty \nabla T - \alpha \nabla p$ . In this case the expression for the production of entropy has the form

$$\sigma_s = \lambda_\infty \frac{\nabla T \cdot \nabla T}{T^2} + \alpha \frac{\nabla T \cdot \nabla p}{T^2} \geq 0. \quad (4)$$

The first term of (4) is always positive, and the second is also always positive, since  $\nabla p$  and  $\nabla T$  are taken in the same direction ( $\nabla p = \alpha \nabla T$ ) and  $\alpha \frac{\nabla T \cdot \nabla p}{T^2} = \alpha \alpha \frac{\nabla T \cdot \nabla T}{T^2}$ . Hence

$$\sigma_s = (\lambda_\infty + \alpha \alpha) \frac{\nabla T \cdot \nabla T}{T^2} = \lambda^n \frac{\nabla T \cdot \nabla T}{T^2} \geq 0, \quad \lambda^n > \lambda_\infty. \quad (4a)$$

The expression (4a) corresponds to the measurement of thermal conductivity by the nonstationary hot-wire method (Fig. 1).

It should be noted that in [10] Landau and Lifshits discuss the question of the variation of entropy and, in particular, show that in such a discussion it is tacitly assumed that

the heat flux depends only on the temperature gradient and is independent of the pressure gradient. This assumption is not evident *a priori* and may be justified as follows. If  $\vec{I}_q$  contained a term proportional to  $\nabla p$ , then a further term proportional to  $\nabla p \cdot \nabla T$  would be added to the expression for the change in entropy. Since this latter product may be either positive or negative, the change in entropy would not be essentially positive. For systems whose volume is constant,  $\sigma_s \geq 0$ , since  $\nabla p$  and  $\nabla T$  are taken in the same direction.

When the volume of the system varies, the corresponding thermodynamic forces can be written in the form

$$\vec{X}_q = -\frac{5}{3} \frac{\nabla T}{T^2} + \frac{2}{3} \frac{\nabla p}{\rho T}, \quad \vec{X}_M = \frac{k}{M} \left( \frac{3}{2} \nabla \ln T - \nabla \ln \rho \right)$$

and the expression for the effective thermal conductivity has the form

$$\lambda^H = \lambda_\infty \left( 1 - \frac{2}{3} \frac{\nabla \ln \rho}{\nabla \ln T} + \tilde{Q}^* \frac{\nabla \ln \rho}{\nabla \ln T} \right), \quad (5)$$

where

$$\lambda_\infty = \frac{L_{22}}{T^2} \left( \frac{5}{3} - \frac{3}{2} \tilde{Q}^* \right).$$

This corresponds to the case in which the thermal conductivity is measured by the nonstationary shock-tube method. In the experimental process for determining the thermal conductivity of a gas by the nonstationary shock-tube method, we have variation in the temperature and density of the gas, while the pressure remains practically constant. Since in this case  $\nabla \ln \lambda = -\nabla \ln T$ , the relation (5) reduces to the expression  $\lambda^H = \lambda_\infty (1 - \tilde{Q}^*)$ .

Thus,  $\lambda^H < \lambda_\infty$ , which is confirmed by the experimental data given in Fig. 2. Practically all data on the thermal conductivity of gases that are obtained with shock tubes [11-13] show that  $\lambda^H$  is always less than  $\lambda_\infty$ .

The expression for the heat flux has the form

$$\vec{I}_q = -\lambda_\infty \nabla T - \lambda \nabla \rho,$$

and the expression for the production of entropy has the form

$$\sigma_s = \lambda_\infty \frac{\nabla T \cdot \nabla T}{T^2} + \lambda \frac{\nabla T \cdot \nabla \rho}{T^2} = (\lambda_\infty - a\lambda) \frac{\nabla T \cdot \nabla T}{T^2} \geq 0.$$

It should be noted that  $a\lambda < \lambda_\infty$ , since  $\lambda$  characterizes a second-order crossover effect (heat transfer caused by the density gradient). Here  $\lambda^H = \lambda_\infty - a\lambda$  and  $\lambda^H < \lambda_\infty$ .

In [14], when the change in entropy is considered, the coefficient  $\lambda$  in the relation  $\vec{I}_q = -\lambda_\infty \nabla T - \lambda \nabla \rho$  is at once set equal to zero, in order to make sure that the term proportional to  $\nabla \rho \cdot \nabla T$  in the expression for the production of entropy will be nonnegative. It should be noted that this assumption corresponds to the case of stationary thermal conductivity.

The above analysis shows that the lower values of data on the thermal conductivity of gases which are obtained by the nonstationary shock-tube method are due essentially to the contribution made by the density gradient to the total heat flux. This reduction should be classified as one of the components of the error in the method, which may reach values of -10% to -14% [6-8].

#### NOTATION

$\vec{I}_q$ , heat flux;  $L_{21}$ ,  $L_{22}$ , phenomenological coefficients;  $\vec{X}_M$ ,  $\vec{X}_q$ , thermodynamic forces;  $\lambda^H$ ,  $\lambda_\infty$ , effective thermal conductivities measured by nonstationary and stationary methods;  $p$ , pressure;  $T$ , temperature;  $M$ , mass;  $\sigma_s$ , production of entropy;  $\rho$ , density;  $k$ , Boltzmann constant.

#### LITERATURE CITED

1. J. J. De Groot, J. Kestin, and H. Sookiazian, "Instrument to measure the thermal conductivity of gases," *Physica*, 75, 454-470 (1975).

2. A. A. Clifford, J. Kestin, and W. A. Wakeham, "Thermal conductivity of N<sub>2</sub>, CH<sub>4</sub>, and CO<sub>2</sub> at room temperature and pressure up to 35 MPa," *Physica*, 97A, No. 3, 287-295 (1979).
3. J. W. Haarman, Een nauwkeurige methode voor het bepalen van de warmtegeleidings-coëfficiënt van gassen (De niestationaire draadmethode), Delft (1969), p. 250.
4. I. Mashtovskii, "Thermal conductivity of mixtures of helium and xenon at high temperatures," *Inzh.-Fiz. Zh.*, 32, No. 4, 635-641 (1977).
5. D. I. Collins, R. Greif, and A. E. Bryson, "Measurements of the thermal conductivity of helium in the temperature range 1600-6700°K" *Int. J. Heat Transfer*, 8, 1209-1216 (1965).
6. N. B. Vargaftik and Yu. D. Vasilevskaya, "Thermal conductivity of krypton and xenon at high temperatures up to 5000°K," *Inzh.-Fiz. Zh.*, 39, No. 5, 852-858 (1980).
7. N. B. Vargaftik and Yu. D. Vasilevskaya, "High-temperature thermal conductivity of neon up to 5000°K and argon up to 6000°K," *Inzh.-Fiz. Zh.*, 40, No. 3, 473-481 (1981).
8. N. B. Vargaftik and Yu. D. Vasilevskaya, "Thermal conductivity of helium at temperatures of 300-6000°K," *Inzh.-Fiz. Zh.*, 42, No. 3, 412-417 (1982).
9. S. R. De Groot, *Thermodynamics of Irreversible Processes* [Russian translation], GITTL, Moscow (1956).
10. L. D. Landau and E. M. Lifshits, *Mechanics of Continuous Media* [in Russian], GITTL, Moscow (1953).
11. D. I. Collins and V. A. Menard, "Measurement of the thermal conductivity of inert gases in the temperature range from 1500 to 5000°K," *Teploperedacha, Ser. C*, No. 1, 56-59 (1966).
12. E. F. Smiley, *The Measurement of the Thermal Conductivity of Gases at High Temperatures with Shocktube; Experimental Results in Argon at Temperatures between 1000°K and 3000°K*, Ph. D. thesis, The Catholic University of America (1957), Washington, p. 132.
13. R. A. Matula, "Thermal conductivity of rarefied gases and gas mixtures at high temperatures," *Teploperedacha, Ser. C*, No. 3, 40-49 (1968).
14. R. C. Balescu, *Equilibrium and Non-Equilibrium Statistical Mechanics*, Wiley (1975).
15. N. B. Vargaftik, L. P. Filippov, A. A. Tarzimanov, and E. E. Totskii, *Thermal Conductivity of Liquids and Gases* [in Russian], Standartov, Moscow (1978).

#### MEASUREMENT OF PULSATIONS IN HEAT FLOW ON THERMALLY LOADED SURFACES

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Pulsations in heat flow on a surface, washed by a fluidized bed, are measured with the help of a sensor.

Various methods are used to measure the heat flow and heat transfer coefficient in experimental studies of external heat transfer in a fluidized bed. In order to determine the time-averaged heat transfer coefficient, massive calorimeters are used with different configurations: spherical, cylindrical, and flat. Heat transfer is calculated starting from the solution of the heat conduction equation for a sensing body (local values) by the regular thermal regime method or the balance method, namely, by the electrical power supplied to the heater of the calorimeter (surface-averaged values).

In order to find the instantaneous values of the coefficient of heat transfer, constant current thermoanemometers are widely used. First used only for a qualitative verification of the nonstationary nature of the external heat exchange in the bed [1] (after refining the measuring procedure), they were then used to obtain a quantitative description of the process [2, 3]. The equation for  $\alpha$  was based on the equation of heat balance of the foil in the thermoanemometer. Calculations using this equation, as shown in [4], could only give a heat transfer coefficient averaged over a half period of the oscillations. Analysis of the nonstationary temperature field of the substrate, performed in [4], permitted eliminating in the calculation such quantities as the effective heat capacity of the foil and the heat loss in